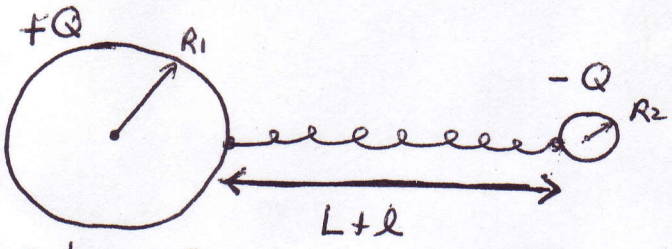
Name:

# Physics 102 Test #1

2) A fixed sphere of radius $R_1$ is attached, by a spring, to a sphere of radius $R_2$ that is free to move. The fixed sphere has charge $+Q$ and the free sphere has charge $-Q$ and mass $m$. The spring has an equilibrium length $L$ and a spring constant $k$. The spring is initially stretched by a length $\ell$ and the free sphere is released from rest. Find the speed of the free sphere when it passes through the equilibrium position of the spring. (6 marks)

Initial State :




This sphere is fixed and does not move

Use Conservation of Energy. (1)

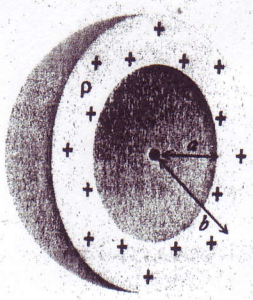
$$E_i = \frac{1}{2}k\ell^2 - \frac{k_e Q^2}{L+\ell+R_1+R_2}$$

$$E_f = \frac{1}{2}mv^2 - \frac{k_e Q^2}{L+R_1+R_2}$$ (2)

$$E_i = E_f \Rightarrow \frac{1}{2}mv^2 - \frac{k_e Q^2}{L+R_1+R_2} = \frac{1}{2}k\ell^2 - \frac{k_e Q^2}{L+\ell+R_1+R_2}$$

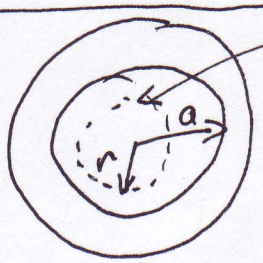
$$\Rightarrow \boxed{v = \sqrt{\frac{2}{m}\left[\frac{1}{2}k\ell^2 + \frac{k_e Q^2}{L+R_1+R_2} - \frac{k_e Q^2}{L+\ell+R_1+R_2}\right]}}$$

1) A **nonconducting** spherical shell of inner radius $a$ and outer radius $b$ has a uniform charge density $\rho$ distributed over its volume (i.e. in the region $a \le r \le b$). Find the electric field in the regions $r < a$, $a \le r \le b$ and $r > b$. (10 marks)




use Gauss' Law $\oint \vec{E} \cdot d\vec{a} = q_{in}/\varepsilon_0$ (1)

$r<a$



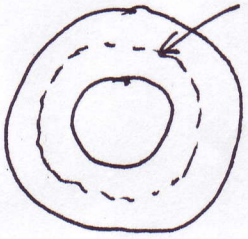

Gaussian Sphere (1)

$\oint \vec{E} \cdot d\vec{s} = AE = 4\pi r^2 E$ (1)

$q_{in} = 0$

$\Rightarrow 4\pi r^2 E = 0 \Rightarrow \boxed{\vec{E} = 0 \quad r<a}$ (1)

$a \le r \le b$



Gaussian sphere

$\oint \vec{E} \cdot d\vec{a} = 4\pi r^2 E$

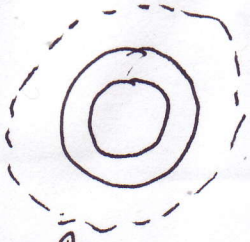
$$q_{in} = \int_a^r \rho (4\pi r^2 dr) = 4\pi\rho \int_a^r r^2 dr$$
$$= 4\pi\rho \left[\frac{1}{3} r^3\right]_a^r = \frac{4\pi}{3}\rho (r^3 - a^3)$$ (2)

or $\rho V$ ← no int. nec.

$$\Rightarrow 4\pi r^2 E = \frac{4\pi}{3\varepsilon_0}\rho (r^3 - a^3) \Rightarrow \boxed{\vec{E} = \frac{1}{3\varepsilon_0}\rho \left(r - \frac{a^3}{r^2}\right)\hat{r}} \quad a \le r \le b$$ (1)

$r \ge b$ $\oint \vec{E} \cdot d\vec{a} = 4\pi r^2 E$

$$q_{in} = \int_a^b \rho (4\pi r^2 dr) = 4\pi\rho \left[\frac{1}{3} r^3\right]_a^b$$
$$q_{in} = \frac{4\pi}{3}\rho \left[b^3 - a^3\right] \; (= Q)$$ (2)

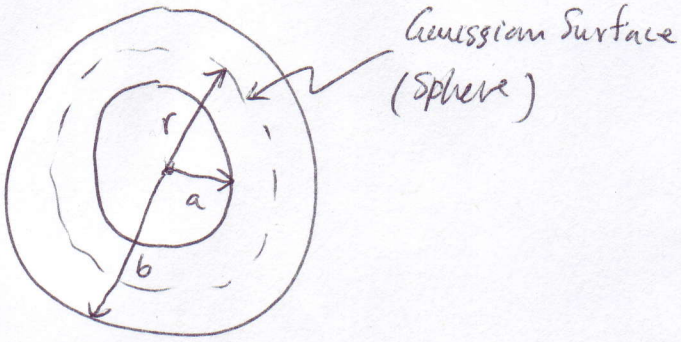


Gaussian Sphere

$$\boxed{\vec{E} = \frac{\rho (b^3 - a^3)}{3\varepsilon_0 r^2}\hat{r}}$$ (1) $r > b$

Alternate soln to 1. for $a \le r \le b$

$a \le r \le b$




$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = E\, 4\pi r^2$$

volume of gaussian surface — volume of spherical hole.

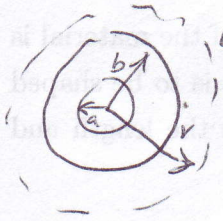
$$q_{inside} = \rho \left[ \frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right] = \frac{4}{3}\pi\rho \left( r^3 - a^3 \right)$$

$$\therefore E\, 4\pi r^2 = \frac{4\pi\rho}{3\varepsilon_0}\left( r^3 - a^3 \right)$$

$$\therefore E = \frac{\rho}{3\varepsilon_0}\left( r - \frac{a^3}{r^2} \right)$$

$\vec{E}$ points radially out. ($\hat{r}$ dir'n)

Alternate sol'n to 1. for $r > b$




$$\oint \vec{E} \cdot d\vec{A} = E\, 4\pi r^2$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0}$$

$$q_{inside} = \rho \left[ \frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 \right]$$

$$= \frac{4\pi}{3}\rho \left( b^3 - a^3 \right)$$

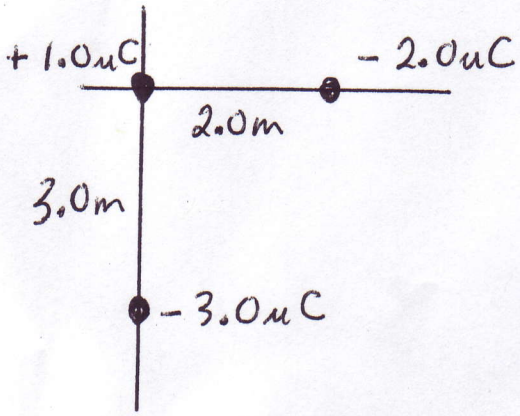
$$\therefore E = \frac{\rho}{3\varepsilon_0 r^2}\left(b^3 - a^3\right)$$

$\vec{E}$ pts radially outward $(\hat{r})$

## Multiple Choice Questions (2 marks each)

1) Three charges are arranged as shown below. What is the net electric force on the $-2.0\ \mu C$ charge?

A) $-2.2 \times 10^{-3}\hat{\imath} + 3.5 \times 10^{-3}\hat{\jmath}$ N
B) $-1.2 \times 10^{-3}\hat{\imath} + 7.6 \times 10^{-3}\hat{\jmath}$ N
C) $-4.1 \times 10^{-3}\hat{\imath} + 1.9 \times 10^{-3}\hat{\jmath}$ N
D) $-3.7 \times 10^{-3}\hat{\imath} + 5.4 \times 10^{-3}\hat{\jmath}$ N
E) $-5.8 \times 10^{-3}\hat{\imath} + 6.7 \times 10^{-3}\hat{\jmath}$ N




Answer: A

2) If the potential is given by $V = 2xy - 3x^2y^2$ what is the electric field at the point $(x, y) = (2, 1)$ ?

A) $4\hat{\imath} - 8\hat{\jmath}$ N/C
B) $15\hat{\imath} + 30\hat{\jmath}$ N/C
C) $2\hat{\imath} - 4\hat{\jmath}$ N/C
D) $5\hat{\imath} - 10\hat{\jmath}$ N/C
E) $10\hat{\imath} + 20\hat{\jmath}$ N/C

Answer: E